

FORMATION OF PDE \rightarrow

The PDE can be formed using the following methods.

(i) By elimination of arbitrary constants.

(ii) By elimination of arbitrary functions.

Problems :-

$$(1) \quad 2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

$$(2) \quad z = a(x+y) + b(x-y) + abt + c$$

$$(3) \quad (x-a)^2 + (y-b)^2 + z^2 = c^2$$

$$(4) \quad z = f(y/x)$$

$$(5) \quad z = f(x+it) + g(x-it)$$

$$(6) \quad z = y^2 + 2f(1/x + \log y)$$

$$(7) \quad z = x^2 f(y) + y^2 g(x)$$

Q. Form PDE by eliminating the arbitrary

constants $z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ — (1)

Sol. Differentiating partially (1) w.r.t x ,

$$2 \frac{\partial z}{\partial x} = \frac{2x}{a^2} \Rightarrow \frac{\partial z}{\partial x} = \frac{x}{a^2} \Rightarrow \frac{1}{a^2} = \frac{1}{2} \frac{\partial z}{\partial x} = \frac{p}{2}$$

Again differentiating partially (1) w.r.t y ,

$$2 \frac{\partial z}{\partial y} = \frac{2y}{b^2} \Rightarrow \frac{\partial z}{\partial y} = \frac{y}{b^2} \Rightarrow \frac{1}{b^2} = \frac{1}{2} \frac{\partial z}{\partial y} = \frac{q}{2}$$

eqⁿ. (1) becomes,

$$z = x^2 \left(\frac{p}{2} \right) + y^2 \left(\frac{q}{2} \right)$$

$$z = xp + yq$$

which is the reqd. PDE.

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